shows the skin friction distribution along the wall and Fig. 3 shows the growth of the velocity profile along the generatrix of the cone (a-d correspond to $\xi = 0$, 10^{-2}, $1.2 \cdot 10^{-2}$, and $3 \cdot 10^{-2}$). The absorption takes place very rapidly by the length scale L. In conclusion, it is worth noting that, in the general case, the displacement process will be accompanied by chemical reactions and, strictly speaking, the entire analysis given above is valid only when the Lewis number Le = 1.

LITERATURE CITED

1. A. P. Komashenko and V. Ya. Neiland, "Hypersonic flow past sphere in the presence of massive injection from its surface," Dokl. Akad. Nauk Ukr. SSR, No. 12 (1969).
2. É. A. Gershbein and G. A. Tirskii, "Viscous flow of conducting, multicomponent gas in the shock layer near a blunt nose with massive injection," in: Proc. of the Institute of Mechanics [in Russian], No. 1, Moscow State Univ. (1970).
3. E. A. Gershbein, "Theory of viscous hypersonic shock layer at large Reynolds numbers and massive injection of extraneous gases," Prikl. Mat. Mekh., No. 6 (1974).
4. É. A. Gershbein, "Asymptotic study of three-dimensional hypersonic flow of viscous fluid past blunt bodies with porous surface," in: Three-Dimensional Hypersonic Flow in the Presence of Physicochemical Reactions [in Russian], Moscow State Univ. (1981).
5. G. G. Chernyi, Hypersonic Flow Theory [in Russian], Fizmatgiz, Moscow (1959).
6. V. Ya. Neiland, "Solutions to laminar boundary-layer equations with arbitrary initial conditions," Prikl. Mat. Mekh., 30, No. 4 (1966).

UNSTEADY MOTION OF A CIRCULAR CYLINDER IN A TWO-LAYER LIQUID

V. I. Bukreev, A. V. Gusev, and I. V. Sturova

UDC 523.593

We have performed a theoretical and experimental study of plane internal waves generated on the interface in a two-layer liquid by the unsteady translational motion of a submerged circular cylinder. At the present time wave formaton by such motion of a two-dimensional body has been analyzed theoretically only for the special case of a homogeneous liquid [1], and experimental research has been devoted mainly to the study of steady motion [2, 3].

We consider the linear formulation of the two-dimensional problem of wave flows generated by a dipole with a time-dependent moment moving in the upper layer of a two-layer liquid. We assume that, as in an infinite homogeneous liquid, this is equivalent to the motion of a circular cylinder of radius R with the velocity U(t) [the dipole moment $m(t) = 2\pi R^2 U(t)$, and coincides with the direction of motion of the cylinder]. We assume that the liquid is inviscid and incompressible, and consists of two layers of different densities: $\rho_1 (0 < y < H_1)$ and $\rho_2 = \rho_1(1 + \varepsilon)$, $\varepsilon > 0 (-H_2 < y < 0)$. The y axis is directed vertically upward, and the horizontal x axis lies in the undisturbed interface. We assume that at time t = 0 a dipole with the variable moment m(t) [$m(t) \equiv 0$ for $t \leq 0$] with its axis in the positive direction of the x axis begins to act in the upper layer of liquid at the point x = 0, y = h, so that its trajectory has the form x = c(t), y = h.

We assume potential flow in each layer, and that the equations of motion have the form

$$\Delta v_n = -\gamma_n m(t) \frac{\partial}{\partial x} \delta(x - c(t)) \frac{\partial}{\partial y} \delta(y - h)$$

with the boundary conditions

$$v_1 = 0 \quad \text{at} \quad y = H_1,$$

$$v_1 = v_2, \left[\rho_n \left(\frac{\partial^3 v_n}{\partial t^2 \partial y} - g \frac{\partial^2 v_n}{\partial x^2}\right)\right]_2^1 = 0 \quad \text{at} \quad y = 0,$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 101-106, November-December, 1983. Original article submitted July 9, 1982.

0021-8944/83/2406-0856$07.50

$$v_2 = 0 \quad \text{at} \quad y = -H_2, \quad v_n \to 0 \quad \text{at} \quad |x| \to \infty$$

and the initial conditions

$$v_n = \partial v_n/\partial t = 0 \quad \text{at} \quad t = 0.$$

Here v_n is the vertical component of the velocity, g is the acceleration due to gravity, n = 1, 2 (the subscript 1 refers to the upper layer, and 2 to the lower layer), $\gamma_1 = 1$, $\gamma_2 = 0$. The condition of a "solid cover" on the free surface ($y = H_1$) permits filtering out the surface waves without substantially distorting the waves on the interface, and simplifies the solution of the problem somewhat.

Taking the Fourier transform with respect to x and the Laplace transform with respect to t,

$$\bar{v}_n(k, y, s) = \int_0^\infty e^{-st} dt \int_{-\infty}^\infty e^{-ikx} v_n(x, y, t)\, dx$$

for real k and Re s > 0, we obtain for $\bar{v}_n$ the ordinary differential equations

$$\begin{gathered} \bar{v}_n'' - k^2 \bar{v}_n = -\gamma_n \delta'(y - h) P(k, s), \\ \bar{v}_1 = 0 \quad \text{at} \quad y = H_1, \\ \bar{v}_1 = \bar{v}_2,\ s^2 \bar{v}_1' + gk^2 \bar{v}_1 = (1 + \varepsilon)\left(s^2 \bar{v}_2' + gk^2 \bar{v}_2\right) \quad \text{at} \quad y = 0, \\ \bar{v}_2 = 0 \quad \text{at} \quad y = -H_2, \end{gathered} \tag{1}$$

where

$$P(k, s) = ik \int_0^\infty m(\theta) \ \exp\left[-(s\theta + ikc(\theta))\right] d\theta;$$

primes denote differentiation with respect to y.

The study of the behavior of the function $\eta(x, t)$, which describes the elevation of the interface and satisfies the linearized condition $\partial\eta/\partial t = v|_{y=0}$, is of particular interest in the present problem. From the solution of system (1) we obtain an expression for the function $\bar{\eta}(k, s)$ — the double transform of the function η:

$$\bar{\eta}(k, s) = \frac{\bar{v}(k, 0, s)}{s} = s\,(\operatorname{ch} kh - \operatorname{th} kH_1 \operatorname{sh} kh) \operatorname{th} kH_2 \cdot P/D,$$

where $D(s, k) = a(k)s^2 + b(k)$, $a(k) = (1 + \varepsilon) \tanh kH_1 + \tanh kH_2$, $B(k) = \varepsilon gk \tanh kH_1, \tanh kH_2$.

Taking the inverse Fourier and Laplace transforms, we obtain

$$\eta(x, t) = \frac{1}{2\pi} \int_{-\infty}^\infty (\operatorname{ch} kh - \operatorname{th} kH_1 \operatorname{sh} kh) \operatorname{th} kH_2 e^{ikx} F dk,$$

where

$$F = \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{sPe^{st}}{D} ds = \frac{ik}{a} \int_0^t m(\theta) \cos\left[\gamma(t - \theta)\right] e^{-ikc(\theta)} d\theta$$

by virtue of the fact that $a, b \geq 0$, $\gamma(k) = \sqrt{b/a}$.

The final expression for the function η has the form

$$\eta(x, t) = -\frac{1}{\pi} \int_0^\infty \frac{k}{a} \operatorname{th} kH_2 (\operatorname{ch} kh - \operatorname{th} kH_1 \operatorname{sh} kh)\, dk \int_0^t m(\theta) \sin\left[k(x - c(\theta))\right] \cos\left[\gamma(t - \theta)\right] d\theta. \tag{2}$$

This expression is appreciably simplified for an infinitely deep liquid (H_1, $H_2 \to \infty$):

$$\eta_\infty(x,t) = -\frac{1}{\pi(2+\varepsilon)}\int_0^\infty ke^{-kh}dk\int_0^1 m(\theta)\sin[k(x-c(\theta))]\cos\left[\sqrt{gk}(t-\theta)\right]d\theta \tag{3}$$

and is similar to the solution in [1].

The integration with respect to θ in Eqs. (2) and (3) can be performed analytically, in particular for motion of the cylinder at the constant velocity U_0 for a finite time $0 < t < T$. In this case,

$$m(t) = 2\pi U_0 R^2,\ c(t) = U_0 t \quad \text{for} \quad 0 < t \leqslant T,$$
$$m(t) \equiv 0,\ c(t) = U_0 T \quad \text{for} \quad t > T$$

and the inner integral in (2) takes the form

$$\pi U_0 R^2 (f_1/(kU_0+\gamma) + f_2/(kU_0+\gamma)), \tag{4}$$

where, for $0 < t < T$,

$$f_1 = \cos[k(x-U_0t)] - \cos(kx+\gamma t),\quad f_2 = \cos[k(x-U_0t)] - \cos(kx-\gamma t);$$

for $t > T$

$$f_1 = \cos[k(x-U_0T) - \gamma(T-t)] - \cos(kx+\gamma t),$$
$$f_2 = \cos[k(x-U_0T) + \gamma(T-t)] - \cos(kx-\gamma t).$$

To test the validity of the theory presented above for calculating real internal waves, we performed experiments in a noncirculating channel 4.82 m long, 0.2 m wide, and 0.6 m deep filled with two layers of immiscible liquids: water (density $\rho_2 = 1$ g/cm^3, thickness of layer $H_2 = 30$ cm) and kerosene ($\rho_1 = 0.8$ g/cm^3, $H_1 = 15$ cm). Waves were generated by a cylinder of radius $R = 1.03$ cm which was towed in the upper layer at a distance $h < H_1$ from the interface. At time $t = 0$ the cylinder was located at a distance l_0 from one end of the channel. The law of motion of the cylinder $c(t)$ was recorded by resistance transducers. We consider the case of unsteady motion with the velocity of the cylinder described by the expression

$$U(t) = \frac{dc}{dt} = \begin{cases} U_0[1-\exp(-t/t_1)] & \text{for} \quad 0 < t \leqslant T, \\ U_0\exp[-(t-T)/t_2] & \text{for} \quad t > T, \end{cases}$$

where U_0, t_1, t_2, and T are constants.

Wavemeters,† whose operation is based on the substantial difference between the electrical conductivities of water and kerosene, were placed at several fixed distances c_* to record vibrations of the interface. Metrological tests showed that they had a linear static calibration characteristic, negligibly small hysteresis, high sensitivity (about 40 mV/mm), and a space–time resolving power more than adequate for the tests. The output signals of the wavemeters were fed into a recording instrument and also into a specialized HISTOMAT-S computer which analyzed the spectrum of the signals.

In addition to the gravitational waves considered here, capillary waves due to surface tension were generated on the interface between the immiscible liquids, but estimates [4] showed that the capillary waves were negligibly small under the conditions of our experiments.

Unfortunately, there is an important inherent difference between the conditions under which the experiments were performed and calculations based on the linear theory were made. In the calculations the liquids were assumed inviscid, and the layers infinite in the x direction, whereas in the experiments the viscosity of kerosene was $\nu_1 = 0.0162$ cm^2/sec and that of water $\nu_2 = 0.0108$ cm^2/sec, and the channel was of finite length. We discuss below the effect of viscosity on the difference between the experimental and calculated values. Because of the finite length of the channel, waves were reflected from the ends, and seichelike vibra-

†The wavemeters were developed by V. V. Zykov and E. I. Khakhilev.

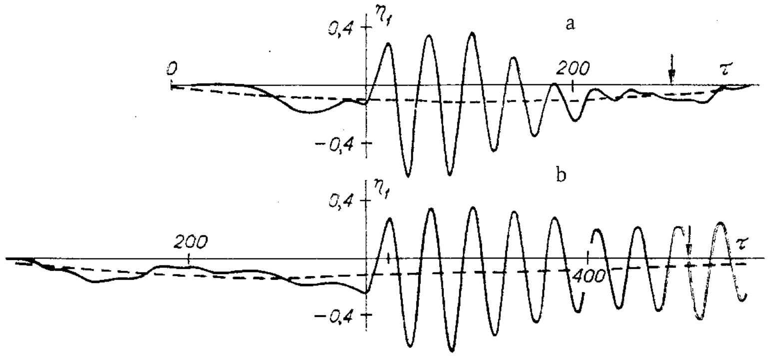

Fig. 1

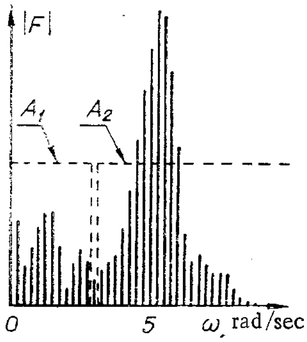

Fig. 2

tions were generated at large times. This limited the time during which the behavior of the dominant modes of vibration of the interface could be studied.

The experiments were performed for a number of values of the controlling parameters of the problem. Here we present only some typical results. The solid lines in Fig. 1 represent the traces of vibrations of the interface in time recorded by two transducers located at distances $c_* = 1$ m (Fig. 1a) and $c_* = 3$ m (Fig. 1b) from the starting point of the cylinder. The values of the other parameters of the problem were identical for these two regimes and equal to the following: $\Lambda = gR/U_0^2 = 2.51$, $h/R = 6$, $\varepsilon = 0.25$, $t_1U_0/R = 3.8$, $t_2U_0/R = 11.4$. The dimensionless time $\tau = tU_0/R$ counted from the start of the motion is plotted along the axis of abscissas, and the dimensionless quantity $\eta_1 = \eta/R$ along the axis of ordinates. On each of the graphs the η_1 axis is displaced to the point corresponding to the instant the cylinder passed over the transducer. The arrows indicate the points on the τ axis corresponding to the instant the cylinder was stopped.

Figure 1 shows that the velocity of the leading edge of the level-lowering wave moving in front of the cylinder is higher than the phase velocity of the main wave train, which is the same as the velocity of the cylinder for uniform motion. Up to the time it reaches the transducer, the level-lowering wave is rather strongly split, and two vibrational modes, which we call long-wave and short-wave, are clearly traced.

We separated the modes mentioned by performing a spectral analysis of the transducer signal. As an example, Fig. 2 shows the modulus of the discrete complex spectrum of the signal $F(\omega)$ corresponding to the solid curve in Fig. 1a. The quantity ω plotted on the axis of abscissas is the circular frequency of the vibrations; the scale along the axis of ordinates is in relative units. The low and high frequencies were separated from the total signal by using filters whose amplitude—frequency characteristics are shown in Fig. 2 by the lines A_1 and A_2.

The long-wave and short-wave modes were found by taking the inverse Fourier transform of the filtered signals. For the example considered here, the long-wave modes are shown by the dashed lines in Fig. 1a, and the short-wave modes by the dashed lines in Fig. 3 (note the different scales on the τ axis in Figs. 1 and 3). The solid lines in Fig. 3 represent the results of calculations based on the linear theory presented above. The integration in (2) was performed numerically. It is of interest to note that for the values of the layer

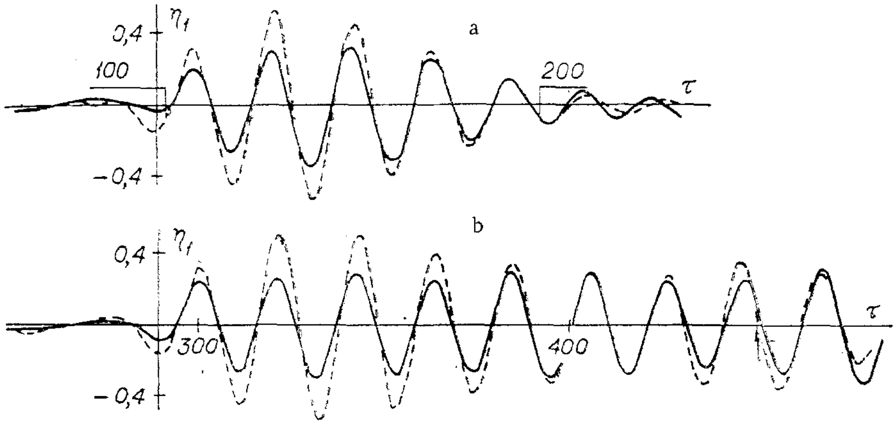

Fig. 3

thicknesses H_1 and H_2 used here there is practically no difference between solutions (2) and (3), so that the layers can be considered infinitely deep.

A comparison of the calculated and experimental data shows that the linear theory describes only the short-wave mode. There is good agreement between the phase pattern of the vibrations, but a certain difference in amplitude.

The difference between the calculated and experimental data due to viscosity manifests itself in various ways. First of all, the viscosity affects the pattern of flow around the body, and leads to the formation of a boundary layer and a wake. For steady motion an impulsive force equal to the resistance of the body acts on the liquid. For unsteady motion an intense initial vorticity is also introduced into the flow. Since the theory of potential flow around a body does not include these effects, there is a difference between the calculated and experimental data for the long-wave mode and the initial amplitude of the short-wave mode.

Another effect of viscosity which manifests itself in the damping of internal waves can be taken into account within the framework of the linear theory (at least in a qualitative way) by retaining the term with the viscosity in the initial equations. The corresponding calculations and their experimental verification were performed in [4]. It was found that the damping is exponential, so that this effect of viscosity can be taken into account by multiplying the value of η in the steady case by $\exp(-k_1x)$, where the damping constant k_1 depends on $\varepsilon g/[(2+\varepsilon)U_0^2]$ and $\nu_{1,2}$, but not on h and the shape of the body.

Generalizing this result, we propose the following semiempirical method for taking account of the effect of viscosity not only on the damping of waves, but also on the initial disturbance: The value of η found by linear wave theory in an ideal liquid must be multiplied by $a_1 \exp(-k_1x)$, where a_1 depends also on the shape of the body. The parameters a_1 and k_1 are best found from experiment. For the graphs in Fig. 3 good agreement with the experimental data is achieved for $a_1/R = 2$ and $k_1R = 5 \cdot 10^{-3}$. For waves generated by the motion of a streamlined wing, a_1/R is much closer to unity, i.e., there is better agreement with the linear theory [3]. Thus, in the unsteady case only the short-wave mode can be corrected in this way.

The effect of viscosity is also related to the appearance of drift flows, a certain curving of the interface, partially compensating the resistance, and other effects which are less important those those considered above. These can also explain a certain increase of the amplitude of the short-wave mode at large τ in Fig. 3b.

In conclusion, we consider the question of whether it is impossible to replace the actual law of motion of the cylinder (5) by the step function

$$U(t)=\begin{cases}U_0 & \text{for } 0<t\leqslant T,\\ 0 & \text{for } t>T.\end{cases}$$

This question was analyzed by comparing the results of calculations with Eqs. (2) and (4). It was found that for the above values of the parameters t_1 and t_2 this replacement leads only to the displacement of the wave train as a whole along the τ axis by a constant amount close to t_1U_0/R.

LITERATURE CITED

1. L. N. Sretenskii, The Theory of Wave Motions of a Fluid [in Russian], Nauka, Moscow (1977).
2. N. Salvesen, "On higher-order wave theory for submerged two-dimensional bodies," J. Fluid Mech., 38, 415 (1969).
3. V. I. Bukreev, "Waves generated at an interface between two fluids of different densities by the motion of a circular cylinder and a symmetric wing," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1980).
4. N. N. Borodina, V. I. Bukreev, A. V. Gusev, and I. V. Sturova, "Viscous damping of internal waves generated in a two-layer liquid by the motion of a cylinder and wing," in: Mathematical Problems of the Mechanics of Continuous Media [in Russian], No. 54, Inst. Hydrodynamics, Siberian Branch, Academy of Sciences of the USSR, Novosibirsk (1982).

DIFFUSION OF A SPARINGLY SOLUBLE GAS IN A FLOWING LIQUID FILM

P. I. Geshev and A. M. Lapin

UDC 66.071.7:532.59

A film of liquid flowing over an inclined surface is almost always covered by waves, which have a substantial effect on the transfer between phases. The effect of the waves is most pronounced when a sparingly soluble gas is taken up by the film. The increase in the mass-transfer coefficient caused by the waves can be 100% or more [1, 2].

Several factors have been described as responsible for increasing the mass transfer in waves: increase in the overall phase interface area due to the waves (not more than 1% [1]), destruction of the inert gas layer hindering absorption by the waves (although this does not hinder desorption [1]), vortex motion in the gas and liquid phases [2], the presence of vigorous convective flows [3, 4], and complete mixing in the wave depressions [5].

The main resistance to diffusion is localized in the liquid if the gas is sparingly soluble, so diffusion in the gas can be neglected. No vortex motion has been observed up to Reynolds numbers of Re = 200-400 in various experiments on visualizing the flow in films [6, 7], so the intensification is due purely to the wave mechanism [1].

Here we used the thin boundary-layer approximation near the free surface to obtain an exact solution for the diffusion in such a film. It is shown that the main factors that increase the mass transfer for two-dimensional waves are firstly that the surface velocity of the liquid at the wave crests approximates to the phase velocity, which leads to the interesting phenomenon of diffusion independence in the cells of liquid separated by the ridges, and, secondly, the vigorous transverse velocity fluctuations.

Hydrodynamics of a Wave Film. When there are fluctuations in flow rate, one gets two-dimensional waves [1, 8]. The experiments of [1, 8] imply that the wavelength greatly exceeds the average film thickness over a large range in flow rate. The longitudinal-velocity profile is closely approximated by a polynomial of second degree [6]:

$$u(x, y, t)=\frac{3}{2} V(x, t)\left[2 \frac{y}{h(x, t)}-\left(\frac{y}{h(x, t)}\right)^{2}\right], \qquad (1)$$

where V is the film velocity averaged over the cross section; y, normal coordinate reckoned from the wall to the free surface; x, longitudinal coordinate; t, time; and h, instantaneous film thickness.

We take the waves as stationary [i.e., h = h(x − ct), where c is the phase velocity] which gives us hV = hc + const from the condition for constancy of the flow rate in the wave system [9]. We average this over the wavelength to determine the constant as q − <h>c, where q is the average flow rate in the laboratory coordinate system and <h> is the average film thickness. We get for the velocity averaged over the cross section that

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 106-112, November-December, 1983. Original article submitted October 29, 1982.

